

together into sentence meanings, which makes the study of the topics considered in this article an important branch of cognitive psychology.

See also: Eye Movements in Reading; Knowledge Activation in Text Comprehension and Problem Solving, Psychology of; Syntactic Aspects of Language, Neural Basis of; Syntax; Syntax–Semantics Interface; Text Comprehension: Models in Psychology

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Sequential Decision Making

Sequential decision making describes a situation where the decision maker (DM) makes successive observations of a process before a final decision is made, in contrast to dynamic decision making (see *Dynamic Decision Making*) which is more concerned with controlling a process over time.

Formally a sequential decision problem is defined, such that the DM can take observations X_1, X_2, \dots one at a time. After each observation X_n the DM can decide to terminate the process and make a final decision from a set of decisions D , or continue the process and take the next observation X_{n+1} . If the observations X_1, X_2, \dots form a random sample, the procedure is called sequential sampling.

In most sequential decision problems there is an implicit or explicit cost associated with each observation. The procedure to decide when to stop taking observations and when to continue is called the *stopping rule*. The objective in sequential decision making is to find a stopping rule that optimizes the

decision in terms of minimizing losses or maximizing gains including observation costs. The optimal stopping rule is also called the optimal strategy or the optimal policy.

A wide variety of sequential decision problems have been discussed in the statistics literature, including search problems, inventory problems, gambling problems, and secretary-type problems, including sampling with and without recall. Several methods have been proposed to solve the optimization problem under specified conditions, including dynamic programming, Markov chains, and Bayesian analysis.

In the psychological literature, sequential decision problems are better known as *optional stopping problems*. One line of research using sequential decision making is concerned with seeking information in situations such as buying houses, searching for a job candidate, price searching, or target search. The DM continues taking observations until a decision criterion for acceptance is reached. Another line of research applies sequential decision making to account for information processing in binary choice tasks (see *Diffusion and Random Walk Processes; Stochastic Dynamic Models (Choice, Response, and Time)*), and hypothesis testing such as in signal detection tasks (see *Signal Detection Theory: Multidimensional*). The DM continues taking observations until either of two decision criteria is reached. Depending on the particular research area, observations are also called offers, options, items, applicants, information, and the like. Observation costs include explicitly not only possibly money, but also time, effort, aggravation, discomfort, and so on.

Contrary to the objective of statisticians or economists, psychologists are less interested in determining the optimal stopping rule, and more interested in discussing the variables that affect human decision behavior in sequential decision tasks. Optimal decision strategies are considered as normative models, and their predictions are compared to actual choice behavior.

1. Sequential Decision Making with One Decision Criterion

In sequential decision making with one decision criterion the DM takes costly observations X_n , $n = 1, \dots$ of a random process one at a time. After observing $X_n = x_n$ the DM has to decide whether to continue sampling observations or to stop. In the former case, the observation X_{n+1} is taken at a cost of c_{n+1} ; in the latter case the DM receives a net payoff that consists of the payoff minus the observation costs. The DM's objective is to find a stopping rule that maximizes the expected net payoff.

The optimal stopping rule depends on the specific assumptions made about the situation: (a) the dis-

tribution of X is known, not known or partly known, (b) X_i are distributed identically for all i , or have the similar distribution but with different parameters, or have different distributions, (c) the number of possible observations, n , is bounded or unbounded, (d) the sampling procedure, e.g., it is possible to take the highest value observed so far when stopping (sampling with recall) or only to take the last value when stopping (sampling without recall), and (e) the cost function, c_n , is fixed for each observation or is a function of n . Many of these problems have been studied theoretically by mathematicians and experimentally by psychologists. Pioneering experimental work was done in a series of papers by Rapoport and colleagues (1966, 1969, 1970, 1972).

1.1 Unknown Sample Distribution: Secretary-type Problems

Kahan et al. (1967) investigated decision behavior in a sequential search task where the DM had to find the largest of a set of $n = 200$ numbers, observed one at a time from a deck of cards. The observations were taken in random order without replacement. The DM could only declare the current observation as the largest number (sampling without recall), and could compare the number with the previous presented numbers. No explicit cost for each observation was taken, i.e., $c = 0$. The sample distribution was unknown to the DM. A reward was paid only when the card with the highest number was selected, and nothing otherwise. This describes a decision situation that is known as *the secretary problem* (a job candidate search problem; for various other names, see, e.g., Freeman 1983) which, in its simplest form, makes explicit the following assumptions (Ferguson 1989): (a) only one position is available, (b) the number n of applicants is known, (c) applicants are interviewed sequentially in random order, each order being equally likely, and (d) all applicants can be ranked without ties—the decision to reject or accept an applicant must be based only on the relative ranks of the applicants interviewed so far, (e) an applicant once rejected cannot later be recalled, and (f) the payoff is 1 when choosing the best of the n applicants, 0 otherwise.

The optimal strategy for this kind of problem is to reject the first $s-1$, $s \geq 1$, items (cards, applicants, draws) and then choose the first item that is best in the relative ranking so far. With

$$a_s \equiv \frac{1}{s} + \frac{1}{s+1} + \dots + \frac{1}{n-1} \quad (1)$$

the optimal strategy is to stop if $a_s < 1$ and to continue if $a_s > 1$, which can easily be determined for small n . For large n , the probability of choosing the best item is

approximated by $1/e$ and the optimal s by n/e . ($e = 2.71\dots$). (For derivations, see, e.g., DeGroot 1970, Freeman 1983, Gilbert and Mosteller 1966).

Kahan et al. (1967) reported that about 40 percent of their subjects did not follow the optimal strategy but stopped too late and rejected a card that should have been accepted. The failure of the strategy for describing behavior was assigned to its inadequacy for the described task. Although at the beginning of the experiment the participants did not know anything about the distribution (requirement), they could learn about the distribution by taking observations (partly information). To guarantee ignorance of the distribution, Gilbert and Mosteller (1966) recommended supplying only the rank of the observation made so far and not the actual value. Seale and Rapoport (1997) conducted an experiment following this advice. They found that participants (with $n = 40$ and $n = 80$) stopped earlier than prescribed by the optimal stopping rule. They proposed simple decision rules or heuristics to describe the actual choice behavior. Using a cutoff rule, the DM rejects the first $s - 1$ applicants and then chooses the next top-ranked applicant, i.e., the candidate. The DM simply counts the number of applicants and then stops on the first candidate after observing $s - 1$ applicants. Under a candidate count rule, the DM counts the number of candidates and chooses the j th candidate. A successive non-candidate rule requires the DM to choose the first candidate after observing at least k consecutive noncandidates following the last candidate.

The secretary problem has been extended and generalized in many different directions within the mathematical statistics field. Each of the above assumptions has been relaxed in one way or another. (Ferguson 1989, Freeman 1983). However, the label of secretary problem tends to be used only when the distribution is unknown and the decision to stop or to continue depends only on the relative ranking of the observations taken so far and not on their actual values.

1.2 Known Sample Distribution

Rapoport and Tversky (1966, 1970) investigated choice behavior when the mean and the variance of the distribution was known to the DM. The cost for each observation was fixed but the amount varied across experimental conditions, and the number of possible observations n was unbounded (1966) or bounded and known (1970). Behavior for sampling with and without recall was compared. When sampling is without recall only the value of the last observation, $X_n = x_n$, can be received, and the payoff is this value minus the total sampling cost, i.e., $x_n - cn$. The optimal strategy is to find a stopping rule that maximizes the expected payoff $E(X_N - cN)$. When sampling is with recall, the highest value observed so far can be selected and the

payoff is $\max(x_1, \dots, x_n) - cn$ and the optimal strategy is to find a stopping rule that maximizes $E(\max(X_1, \dots, X_N) - cN)$. In the following, v with subscripts and v^* denote the expected gain from an (optimal) procedure.

1.2.1 Number of Observations Unbounded. If n is unbounded, i.e., if the number of observations that can be taken is unlimited, and X_1, X_2, \dots are sampled from a known distribution function $F(x)$, the optimal strategy is the same for both sampling with and without recall. In particular, the optimal strategy is to continue to take observations whenever the observed value $x_j < v^*$, and to stop taking observations as soon as an observed value $x_j \geq v^*$, where v^* is the unique solution of

$$\int_{v^*}^{\infty} (x - v^*) dF(x) = c \quad -\infty < v^* < \infty. \quad (2)$$

When the observations are taken from a standard normal distribution with density functions $\phi(x)$ and distribution function $\Phi(x)$, we have that

$$v^* = \frac{\phi(v^*) - c}{1 - \Phi(v^*)}. \quad (3)$$

Although sampling with and without recall have the same solution, they seem to be different from a psychological point of view. Rapoport and Tversky (1966) found that the group sampling without recall took significantly fewer observations than the participants sampling with recall. The mean number of observations for both groups decreased with increasing cost c , and the difference with respect to the number of observations taken was diminished. However, the participants in both groups took fewer observations than prescribed by the optimal strategy. This nonoptimal behavior of the participants was attributed to a lack of thorough knowledge of the distributions.

1.2.2 Number of Observations Bounded. If $n, n \geq 2$, is bounded, i.e., if not more than n observations can be taken, the optimal stopping rules for sampling with and without recall are different. For sampling without recall, an optimal procedure is to continue taking observations whenever $x_j < v_{n-j} - c$ and to stop as soon as $x_j \geq v_{n-j} - c$, where $j = 1, 2, \dots, n$ indicates the number of observations which remain available and

$$v_{j+1} = (v_j - c) - \int_{v_j - c}^{\infty} (x - (v_j - c)) dF(x). \quad (4)$$

With $v_1 = E(X) - c$, the sequence can be computed successively. Again, assuming a standard normal distribution $v_{j+1} = \phi(v_j - c) + \Phi(v_j - c)$.

For sampling with recall, the optimal strategy is to continue the process whenever a value $x_j < v^*$ and to stop taking observations as soon as an observed value $x_j \geq v^*$, where v^* is as in Eqn. (2), which is the same solution as for n unbounded. (For derivations of the strategies, see DeGroot 1970, Sakaguchi 1961.)

Rapoport and Tversky (1970) investigated choice behavior within this scenario. Sampling was done both with and without recall. The number of observations that could be taken as well as observation cost varied across experimental groups. One third of the participants did not follow the optimal strategy. Under both sampling procedures and all cost conditions, they took on average fewer observations than predicted by the corresponding optimal stopping rules. There were no systematic differences due to cost, as observed in their previous study. They concluded that ‘the optimal model provides a reasonable good account of the behavior of the subjects’ (p. 119).

1.3 Different Sample Distributions for Each Observation

Most research concerned with sequential decision making assumes that the observations are sampled from the same distribution, i.e., X_i are distributed identically for all i . For many decision situations, however, the observations may be sampled from the same distribution family with different parameters, or from different distributions. Especially in economic areas, such as price search, it is reasonable to assume that the distributions from which observations are taken change over time. The sequence of those samples has been called *nonstationary series*. Of particular interest are two special nonstationary series: *ascending* and *descending* series. For ascending series, the observations are drawn from distributions, usually from normal distributions, with increasing mean as i increases; for descending series the mean of the distribution decreases as i increases, i indicating the sample index. For both cases, experiments have been conducted to investigate choice behavior in a changing environment. Shapira and Venezia (1981) compared choice behavior for ascending, descending and constant (identically distributed X_i) series. In one experiment (numbers from a deck of cards), the distributions were known to the DM; no explicit observation costs were imposed; sampling occurred without recall; and the number of observations that could be taken was limited to $n = 7$. The variance of the distributions varied across experimental groups. An optimal procedure was assumed to continue taking observations whenever $x_j < v_{n-j}$, and to stop as soon as $x_j \geq v_{n-j}$, where $j = 1, 2, \dots, n$ indicates the number of

observations which remain available. $k = 1, \dots, n$ indicates the specific distribution for the j th observation. Thus

$$v_{j+1} = v_j - \int_{v_j}^{\infty} (x - v_j) dF_k(x). \quad (5)$$

With $v_1 = E(X_1)$ the sequence can be computed successively. Assuming a standard normal distribution $v_{j+1} = \phi_k(v_j) + \Phi_k(v_j)$.

Across all conditions, 58 percent of the participants behaved in an optimal way. The proportion of optimal stopping did not depend on the type of series but on the size of the variance. Nonoptimal stopping (24 percent stopped too early; 18 percent too late) depended on the series and on the size of the variance. In particular, participants stopped too early on ascending and too late on descending series. A similar result was observed by Brickman (1972). In this study, departing from the optimal stopping rule was attributed to an inadequacy of the stopping rule taken for the particular experimental conditions (assuming complete knowledge of the distributions). In a secretary problem design (see Sect. 1.1), Corbin et al. (1975) were less concerned with optimal choice behavior than with the processes by which the participants made their selections, and with factors that influenced those processes. The emphasis of the investigation was on decision making heuristics rather than on the adequacy of optimal models. With the same optimal stopping rule for all experimental conditions, they found that stopping behavior depended on contextual variables such as the ascending or descending trend of the inspected numbers of the stack.

2. Search Problems—Multiple Information Sources

In a sequential decision making task with multiple information sources, the DM has the option to take information sequentially from different sources. Each information source may provide valid information with a particular probability and at different cost. The task is not only to decide to stop or to continue the process but also, if continuing, which source of information to consult.

Early experimental studies were done by Kanarick et al. (1969), Rapoport (1969), Rapoport et al. (1972). A typical task is to find an object (e.g., a black ball) which is hidden in one of several possible locations (e.g., in one of several bins containing white balls). The optimal search strategy depends on further task specifications, such as whether the object can move from one location to another, how many objects are to be found, and whether the search process may stop before the object has been found. Rapoport (1969)

investigated the case when a single object that could not move was to be found in one of r , $r \geq 2$, possible locations. The DM was not allowed to stop the process before the target was found. All of the following were known to the DM: the a priori probability p_i , $p_i > 0$ that the object is in location i , $i = 1, 2, \dots, r$, with $\sum_{i=1}^r p_i = 1$; a miss probability α_i , $0 < \alpha_i < 1$, that even if the object is in location i it will not be found in a particular search of that location ($1 - \alpha_i$ is referred to the respective detection probability); and a cost, c_i , for a single observation at location i . The objective of the DM is to find a search strategy that minimizes the expected cost. For $i = 1, \dots, r$ and $j = 1, 2, \dots$ let Π_{ij} denote the probability that the object is found at location i during the j th search and the search is terminated. Then

$$\Pi_{ij} = p_i \alpha_i^{j-1} (1 - \alpha_i), \quad i = 1, \dots, r, j = 1, 2, \dots \quad (6)$$

If all values of Π_{ij}/c_i for all values of i and j are arranged in order of decreasing magnitude, the optimal strategy is to search according to this ordering (for derivations, see DeGroot 1970). Ties may be ordered arbitrarily among themselves. The optimal strategy is determined by the detection probabilities and observation costs, and optimal search behavior implies a balance between maximizing the detection probability and minimizing the observation cost. Rapoport (1969) found that participants did not behave optimally. They were more concerned with maximizing the probability of detecting the target than with minimizing observation cost. Increasing the difference of observation cost c_i among the $i = 1, 2, 3, 4$ locations showed that the deviation from the optimal strategy even increased. Rapoport et al. (1972) varied the search problems by allowing the DM to terminate the search at any time; adding a terminal reward, R , for finding the target; and a terminal penalty, B , for not finding the target. Most participants showed a bias toward maximizing detection probability vs. minimizing search cost per observation, similar to the previous study.

3. Sequential Decision Making with Two or More Possible Decisions

A random sample X_1, X_2, \dots is generated by an unknown state of nature, Θ . The DM can take observations one at a time. After observing $X_n = x_n$ the DM makes inferences about Θ based on the values of X_1, \dots, X_n and can decide whether to continue sampling observations or to stop the process. In the former case, observation X_{n+1} is taken; in the latter, the DM makes a final decision $d \in D$. The consequences to the DM depend on the decision d and the value θ .

The statistical theory for this situation was developed by Wald during the 1940s. It has been used to test hypotheses and estimate parameters. In psychological research, sequential decision making of this kind is usually limited to two decisions $D = \{d_1, d_2\}$, and applied to binary choice tasks (see *Diffusion and Random Walk Processes; Stochastic Dynamic Models (Choice, Response, and Time); Bayesian Theory: History of Applications*).

The standard theory of sequential analysis by Wald (1947) does not include considerations of observation costs $C(n)$, losses for terminal decisions $L(\theta, d)$, and a priori (subjective) probabilities π of the alternative states of nature. *Deferred decision theory* generalizes the original theory by including these variables explicitly. The objective of the DM is to find a stopping rule that minimizes expected loss (called risk) and expected observation cost. The form of that optimal stopping rule depends mainly on the assumptions about the number of observations that can be taken (bounded or unbounded), and on the assumption of cost per observation (fixed or not) (see DeGroot 1970). Birdsall and Roberts (1965), Edwards (1965), and Rapoport and Burkheimer (1971) introduced the idea of deferred decision theory as normative models of choice behavior to the psychological community. Experiments investigating human behavior in deferred decision tasks have been carried out by Pitz and colleagues (e.g., Pitz et al. 1969), and by Busemeyer and Rapoport (1988). Rapoport and Wallsten (1972) summarize experimental findings.

For illustration, assume the decision problem in its simplest form. Suppose two possible states of nature θ_1 or θ_2 , and two possible decisions d_1 and d_2 . Cost c per observation is fixed and the number of observations is unbounded. The DM does not know which of the states of nature, θ_1 or θ_2 is generating the observation, but there are a priori probabilities π that it is θ_1 and $(1 - \pi)$ that it is θ_2 . Let w_i denote the loss for a terminal decision incurred by the DM in deciding that θ_i is not the correct state of nature when it actually is ($i = 1, 2$). No losses are assumed when the DM makes a correct decision. Let π_n denote the posterior probability that θ_1 is the correct state of nature generating the observations after n observations have been made. The total posterior expected loss is $r_n = \min\{w_1\pi_n, w_2(1 - \pi_n)\} + nc$. The DM's objective is to minimize the expected loss. An optimal stopping rule is specified in terms of decision boundaries, α and β . If the posterior probability is greater than or equal to α , then decision d_1 is made; if the posterior probability is smaller than or equal to β , then d_2 is selected; otherwise sampling continues.

See also: Decision Making (Naturalistic), Psychology of; Decision Making, Psychology of; Decision Research: Behavioral; Dynamic Decision Making; Multi-attribute Decision Making in Urban Studies

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Sequential Statistical Methods

Statistics plays two fundamental roles in empirical research. One is in determining the data collection process: the experimental design. The other is in analyzing the data once it has been collected. For the purposes of this article, two types of experimental designs are distinguished: *sequential* and *non-sequential*. In a sequential design the data that accrue in an experiment can affect the future course of the experiment. For example, an observation made on one experimental unit treated in a particular way may determine the treatment used for the next experimental unit. The term 'adaptive' is commonly used as an alternative to sequential. In a nonsequential design the investigator can carry out the entire experiment without knowing any of the interim results.

The distinction between sequential and non-sequential is murky. An investigator's ability to carry out an experiment exactly as planned is uncertain, as information that becomes available from within and outside the experiment may lead the investigator to amend the design. In addition, a nonsequential experiment may give results that encourage the investigator to run a second experiment, one that might even simply be a continuation of the first. Considered separately, both experiments are nonsequential, but the larger experiment that consists of the two separate experiments is sequential.

In a typical type of nonsequential design, 20 patients suffering from depression are administered a drug and their improvements are assessed. An example sequential variation is the following. Patients' improvements are recorded 'in sequence' during the experiment. The experiment stops should it happen that at least nine of the first 10 patients, or no more than one of the first 10 patients improve(s). On the other hand, if between two and eight of the first 10 patients improve then sampling continues to a second set of 10 patients, making the total sample size equal to 20 in that case. Another type of sequential variation is when the dose of the drug is increased for the second 10 patients should it happen that fewer than four of the first 10 improve.

Much more complicated sequential designs are possible. For example, the first patient may be assigned a dose in the middle of a range of possible doses. If the patient improves then the next patient is assigned the next lower dose, and if the first patient does not improve then the next patient is assigned the next higher dose. This process continues, always dropping the dosage if the immediately preceding patient improved, and increasing the dosage if the immediately preceding patient did not improve. This is called an 'up-and-down' design.

Procedures in which batches of experimental units (such as groups of 10 patients each) are analyzed before proceeding to the next stage of the experiment