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MDFT account of decision making under time pressure

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In this paper, decision making under time pressure for multiattribute choice alternatives in a risky environment is investigated. A model, multiattribute decision field theory (MDFT), is introduced that describes both the dynamic and the stochastic nature of decision making and accounts for the observed changes in choice probabilities, including preference reversals as a function of time limit. An experiment in which five different time limits were imposed on the decision maker is presented to test the predictions of the model. It is shown that MDFT is able to account for the complex decision behavior observed in the data. Furthermore, MDFT is compared with the predictions of decision field theory (Busemeyer & Townsend, 1993; Roe, Busemeyer, & Townsend, 2001).

Research on judgment and decision making under time pressure indicates that the probability of choosing an alternative changes as a function of time constraints (e.g., Edland & Svenson, 1993). In the present paper, an attempt is made to account for this finding by proposing a dynamic stochastic model for decision making under uncertainty and applying it to binary multiattribute choice alternatives.

The approach belongs to a class of models, labeled *sequential comparison models*, in which the decision maker considers features of choice alternatives sequentially over time. Each feature comparison results in a value, which is integrated with the other values, producing a preference state at each moment in time. Preference moves in a random walk manner until a preset decision criterion (or decision boundary), specific to each choice alternative, is reached. At that point, the process stops, and a decision is made (see, e.g., Aschenbrenner, Albert, & Schmalhofer, 1984, for decision making under certainty with multiattribute choice alternatives; see Busemeyer, 1985, 1993; Busemeyer & Townsend, 1992, 1993; and Dror, Busemeyer, & Basola, 1999, for decision making under uncertainty with unidimensional alternatives; see Diederich, 1995, 1996, 1997, for decision making under uncertainty with multiattribute choice alternatives; see Roe, Busemeyer, & Townsend, 2001, for decision making with multiple choice alternatives; see Wallsten & Barton, 1982, for probabilistic inference). Common to these models is that time pressure changes the decision maker's criterion or preference threshold, rather than changing his or her strategies. The criterion is an increasing function of the time available. These approaches explicitly model the process until one reaches a decision, and therefore, they naturally yield predictions for decision making under time constraints.

First, I will briefly present a dynamic stochastic decision model for binary multiattribute decision problems, called *multiattribute decision field theory* (MDFT), and its specific predictions for decision making under time constraints. Details—in particular, mathematical derivations—can be found in Diederich (1997) and Busemeyer and Diederich (2002). MDFT extends and generalizes the decision field theory (DFT) of Busemeyer and Townsend (1992, 1993), who considered unidimensional choice alternatives. More recently, DFT has been expanded to *multialternative* choice problems, also called MDFT[1] (Roe et al., 2001). They also included the notion of multiattribute alternatives in their model, which will be discussed here briefly. Second, I will report an experiment on decision making under time pressure for multiattribute binary choice alternatives in a risky environment. Five different time limits were imposed on the decision maker, instead of the usual high/low or no/yes time pressure conditions. This allowed us to investigate the change of choice probabilities as a function of time limits more closely. Third, it will be shown that MDFT captures the qualitative trends in the data (e.g., preference reversals). Moreover, it will be shown that MDFT provides a quantitative fit that is a more accurate description of the data than can be provided by DFT.

MULTIATTRIBUTE DECISION FIELD THEORY

According to MDFT, decisions are based on preference states, $P(t)$, representing the momentary preference for choosing one alternative over the other in a binary choice—for instance, A over B. That is, $P(t)$ reflects the relative strength of preference of choosing A over B at time t. More specifically, a positive value of $P(t)$ represents a preference strength in the direction of favoring A, whereas a

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negative value of $P(t)$ represents a preference strength in the direction of favoring B. Preference evolves over time and is updated by an input valence, reflecting the momentary comparison of consequences produced by imaging the choice of either A or B. The valence fluctuates because the decision maker's attention switches back and forth between anticipated consequences. For multiattribute alternatives, it is assumed that the preference process has a specific input valence, $V_i(t)$, for each attribute comparison. Technically, $V_i(t)$ is a random variable that is able to account for the decision maker's fluctuation in evaluation of the consequences from one moment to the next. The decision maker draws information about the alternatives and its attributes from his or her memory. The possible consequences connected with either alternative are learned from experience and are remembered more or less well. A decision is made as soon as the preference process reaches a decision criterion or threshold for any of the two alternatives. The dynamics of the preference process is formally described according to the following stochastic process:[2]

$$P(t+1) = P(t) + V_i(t+1).$$

The subscript i refers to the attribute considered at time t. The mean valence for each attribute comparison, denoted $E[V_i(t)] = \delta_i t$, indicates the direction toward choosing A or B. In particular, for $\delta_i > 0$, mean preference is directed toward A, whereas for $\delta_i < 0$, it is directed toward B. (δ is called the *drift rate.*) The process switches (attention shift) from attribute i to attribute j at a particular rate, w_{ij}. For illustration, assume two attributes. At any particular time during deliberation, the decision maker's attention process may be operating on the process for Attribute 1. During the next moment, attention either continues to operate on the process for Attribute 1 with a probability of w_{11}, or attention switches with a probability of $w_{12} = 1 - w_{11}$ to operate on the process for Attribute 2. Similarly, if attention is operating on the process for Attribute 2, then during the next moment, attention may continue to operate on the process for Attribute 2 with probability w_{22} or may switch back to operate on the process for Attribute 1 with a probability of $w_{21} = 1 - w_{22}$. Formally, attention switches from one attribute to another according to a Markov chain process. Roe et al. (2001) assumed that for multiattribute alternatives, the mean valence δ is a weighted average of the attributes, whereas for the present model attention switches according to a mixture of two subprocesses. That is, Roe et al. assumed that attributes are processed in parallel, whereas here it is assumed that attributes are processed in a serial manner. For details, see Busemeyer and Diederich (2002).

The preference process stops, and a decision is initiated as soon as a decision criterion, $\theta > 0$, is reached. If $P(t) > \theta$, A is chosen; if $-P(t) > \theta$, B is chosen. The criterion is assumed to be set by the decision maker prior to the decision task and, among other things, is assumed to depend on the time available for making a decision. Specifically, the criterion boundary is assumed to be an *increasing* function of the time limit. Thus, the probability of choosing A over B is determined by $P(t)$'s reaching the positive threshold before reaching the negative threshold.

Figure 1 illustrates these ideas for two choice alternatives, A and B, with two attributes. Regarding the first attribute, mean valence, δ_1 is larger than 0, indicating a preference for choosing alternative A, whereas for the second attribute δ_2 is smaller than 0 and preference is directed toward B. The trajectories in Figure 1 (left panel) symbolize the preference process evolving over time for each single choice trial. The solid line represents the mean preference over time. $+\theta$ and $-\theta$ are the criteria for choosing A and B, respectively, set by the decision maker.

Assume that the decision maker is put under time pressure. With a short time limit, the decision maker has a low criterion, as indicated by the criterion boundaries θ_1 and

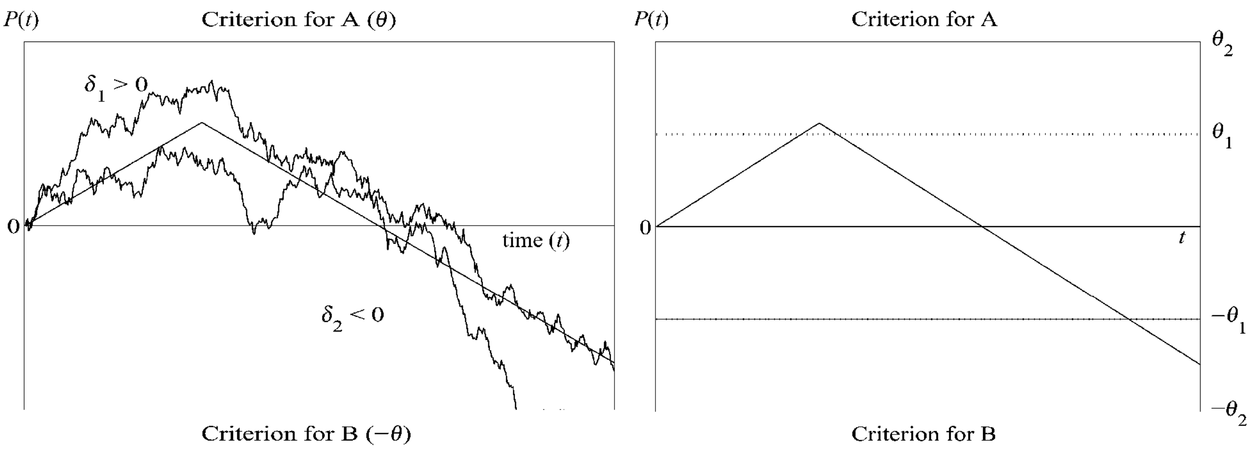

Figure 1. (Left) Hypothetical preference process evolving over time. The trajectories symbolize the preference process evolving over time for each single choice trial. The solid line represents the mean preference (δ) over time. The attributes are processed sequentially. (Right) Mean preference process with decision criteria as a function of time limit. θ_1 and $-\theta_1$ indicate the decision criteria for a short time limit. When more time to decide is available, the decision criterion is assumed to be larger than those for θ_2 and $-\theta_2$.

$-\theta_1$ in Figure 1 (right panel). Thus, the decision criterion is more likely reached by considering the first attribute only, and therefore, the decision maker will more likely choose alternative *A*. With increased time limit, the decision criterion is raised, $+\theta_2$ and $-\theta_2$, and probably the decision maker will also consider the second attribute, and the preference tends toward alternative B.

Note that the deliberation process is neither directly observable nor necessarily open to introspection. However, some properties of the model are obvious (see Figure 1): The preference for alternatives A and B may change during the deliberation process; the preference strength may depend on the particular attribute; the decision may depend on the order in which the attributes are processed; the decision may depend on how long each attribute is processed and, most important, on how much time is available to make a decision.[3]

Predictions of MDFT for Decision Making Under Time Pressure

If it is assumed that, for multiattribute choice alternatives, attributes are considered sequentially, the question arises as to how the attributes are combined to determine the final choice. Several possibilities have been proposed and discussed in detail for alternatives with three attributes (Diederich, 1995, 1996, 1997). One of them will be presented here, exemplified for choice alternatives with two attributes, one of them producing a positive mean valence, the other a negative one. Assume that the decision maker considers a particular attribute, presumably the most salient, first and then switches attention to the next to make a choice. The switching may occur at a given time, or it may occur with a specified probability if little is known about the time point of switching. The latter case will be considered here. For example, let $\delta_1 = .15$ and $\delta_2 = -.1$. Consider two hypothetical attention-switching scenarios: (1) The decision maker switches attention only once, from the first attribute to the second, w_{12}, but not back to the first ($w_{21} = 0$), and (2) attention switches back and forth between both attributes, with $w_{12} = w_{21}$. For simplicity's sake, the switching rate w_{12} is assumed to be time invariant. Figure 2 shows the predicted choice probabilities for (1) and (2) as a function of time limits for the attention-switching rates $w_{12} = .005$, .01, and .05.[4]

Consider first the left panel of Figure 2. With a short time limit, the decision maker more often decides on alternative A (δ_1 has a positive sign)—in particular, when the switching rate is smaller (upper curve). With an increasing time limit, the decision maker more often decides on B, since attention is directed to the second attribute (δ_2 has a negative sign), and the first attribute is not reconsidered. That is, MDFT predicts a preference reversal if the decision maker is put under time pressure and considers attributes with opposite mean valences for the alternatives just once. The right panel of Figure 2 shows the predicted choice probability when the decision maker switches back and forth between both attributes. Overall, there is more evidence of decisions in favor of alternative A (the absolute value of $|\delta_1| = .15$ is larger than the absolute value of $|\delta_2| = .1$). This becomes more evident when the decision maker has more time to make his or her decision—in particular, when the attention-switching rate becomes larger. Note that the switching rate also determines how long each attribute is considered. The decision maker starts considering the first attribute (with $\delta_1 = .15$), and preference is directed toward alternative A. The attention switches to the second attribute, leading to a change of preference toward B. Then, attention switches back to the first attribute, showing more evidence of choosing A. Since with smaller switching rates, each attribute is considered longer than with larger switching rates, the preference for choosing A increases more slowly when the switching rate is small.

Whether we assume attention is switching once or is switching back and forth, preference changes as a func-

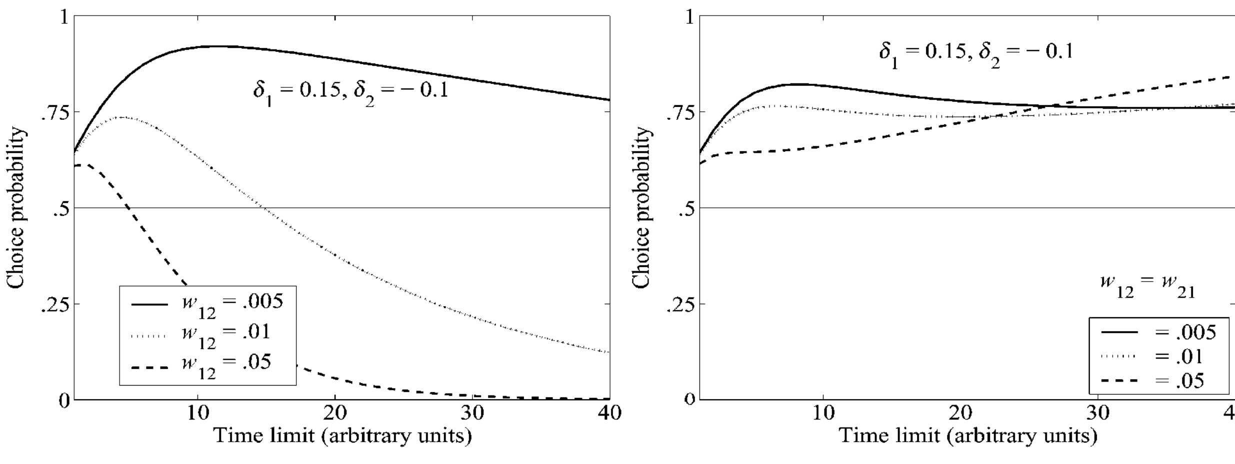

Figure 2. Predicted choice probabilities for choosing alternative A as a function of time limits. For the left panel, it is assumed that the decision maker switches only once, from the first attribute to the second (i.e., $w_{21} = 0$). The right panel refers to the situation in which switching back and forth between both attributes is assumed. Different curves indicate different switching rates. For further details, see the text.

Table 1
Means of Normal Distributions for the Two Attributes of the Four Alternatives

	Alternative			
Attribute	A	B	C	D
Money	−2	−1	1	2
Duration	4	6	10	12

Note—The standard deviation was $\sigma = 2.8$ for money and $\sigma = 1.5$ for duration.

tion of time limit, although in the former case more dramatically than in the latter. Note that both versions of MDFT are able to predict a nonmonotonic relationship between time limits and choice probabilities.

EXPERIMENT

In the following experiment, decision making was investigated under five different time constraints for multiattribute choice alternatives in a risky environment with real consequences. The choice alternatives were described by two attributes: (1) amount of money to win or to lose, and (2) duration of unpleasant sounds (e.g., a dentist's drill) of 69 dB. In each trial, the participant may lose or win money and is forced to listen to the noise for a particular amount of time.

Estimation of choice probability and decision time requires that the same choice alternatives must repeatedly be presented to the participant.[5] In order to minimize the chance that the participants would simply remember their previous choices, the values of the choice alternatives were not identical but were drawn from distributions. The participants learned the distributions, prior to the decision experiment, by feedback (see below).

Method

Design. Four alternatives—labeled A, B, C, and D, all having the two attributes in common—were designed. The specific values of the attributes were drawn from truncated normal distributions and were rounded to integer values. The values drawn from these distributions represented money to gain or to lose (represented by integer numbers; each unit was worth 0.04 Deutsch Mark) and the duration of the noise (in seconds).

Table 1 shows the means of the normal distributions from which the attribute values were drawn. The standard deviations for the money and the noise duration distributions were 2.8 and 1.5, respectively. The parameters were chosen so that alternatives A and B had negative expected values for money but short mean noise duration, whereas alternatives C and D had positive expected values for money but long noise duration. Trials consisted of presenting a pair of alternatives (e.g., A and B), and the participant had to decide which one was the preferred alternative. Across trials, all possible pairs were presented, and the resulting six choice pairs are labeled AB, AC, AD, BC, BD, and CD. Five different time limits (1, 1.5, 2, 2.5, or 3 sec) were imposed on the decision maker, randomly presented during one block of 120 trials.

Participants. Six women and 5 men served as paid voluntary participants in this experiment. They received 10 Deutsch Mark per session, plus the amount of money they had won during each session. One session consisted of two blocks of 120 trials each and lasted for about 1 h.

Procedure. Prior to the decision-making experiment, the participant had to learn the distributions of the attributes. The procedure, however, was the same for the learning session and the decision-making experiment. Each trial began with the display of two letters on a computer screen, representing the choice alternatives and the time limit within which a choice had to be made. The participant was asked to choose one of the two letters by pressing the respective button on a response box. The chosen letter appeared on the screen, and 2 sec later, feedback was delivered; the screen displayed the numerical values of the payoffs produced by each action of that trial. This feedback was followed by the actual delivery of the payoff produced by the letter chosen in that trial. The actual values of the payoffs and the probabilities produced by each action were learned through experience from trial to trial by feedback. The choice for each trial was recorded by the computer. Missing the deadline was punished by a loss of 5 units; each unit was worth 0.04 Deutsch Mark. The noises were presented binaurally via closed headphones; they were generated by a synthesizer and were sampled by a sound card. Each participant performed four blocks of 120 trials each for learning and 24 experimental blocks with 120 trials each. That is, for 30 (five time limits, six different choice pairs) experimental conditions, 3,000 experimental trials were recorded for each participant.

Results and Model Fit

All the participants changed their preference for at least some choice alternatives, depending on the imposed time limit. For all the participants, nonmonotonic relationships between the imposed time limits and the choice probabilities could be observed. Most participants chose the alternative with higher expected value for money more often than the one with lower expected value. (Individual choice patterns, not presented here, showed a preference reversal for choice alternatives depending on time limits. As an example, the data of 1 participant are given in Figure 3.) With extended time limits, the choice probabilities tended to get more extreme—that is, toward 0 (or 1). To see whether preferences for a given choice pair changed significantly as a function of the five time limits, a specific test (called *testing for equality of several probabilities* and described in the Appendix, Table A2) revealed two groups of choice patterns: participants who tended not to change their preferences significantly under time constraints and those who did. Interestingly, it turned out that these groups also differed with respect to gender, and therefore, the data were grouped by gender. (However, investigating gender differences was not the intent of the experiment.)

Women changed their preferences as a function of imposed time limits to a less extent than did the men (see Figure 4, dashes). In fact, testing for changes in preferences as a function of the five time limits for these two groups revealed significant differences for all choice pairs, except for CD, for the male participants, and no significant differences, except for the choice pair AC, for the female participants (see the Appendix, Table A2).

To restrict the model and, thereby, impose a strong restriction on the data, MDFT was fitted to the observed choice patterns under the following assumptions. (1) Attributes were compared and processed serially, and the decision maker considered the most salient attribute first (cf. Wallsten, 1993). In accordance with the probability patterns of both groups, it can be inferred that money was the

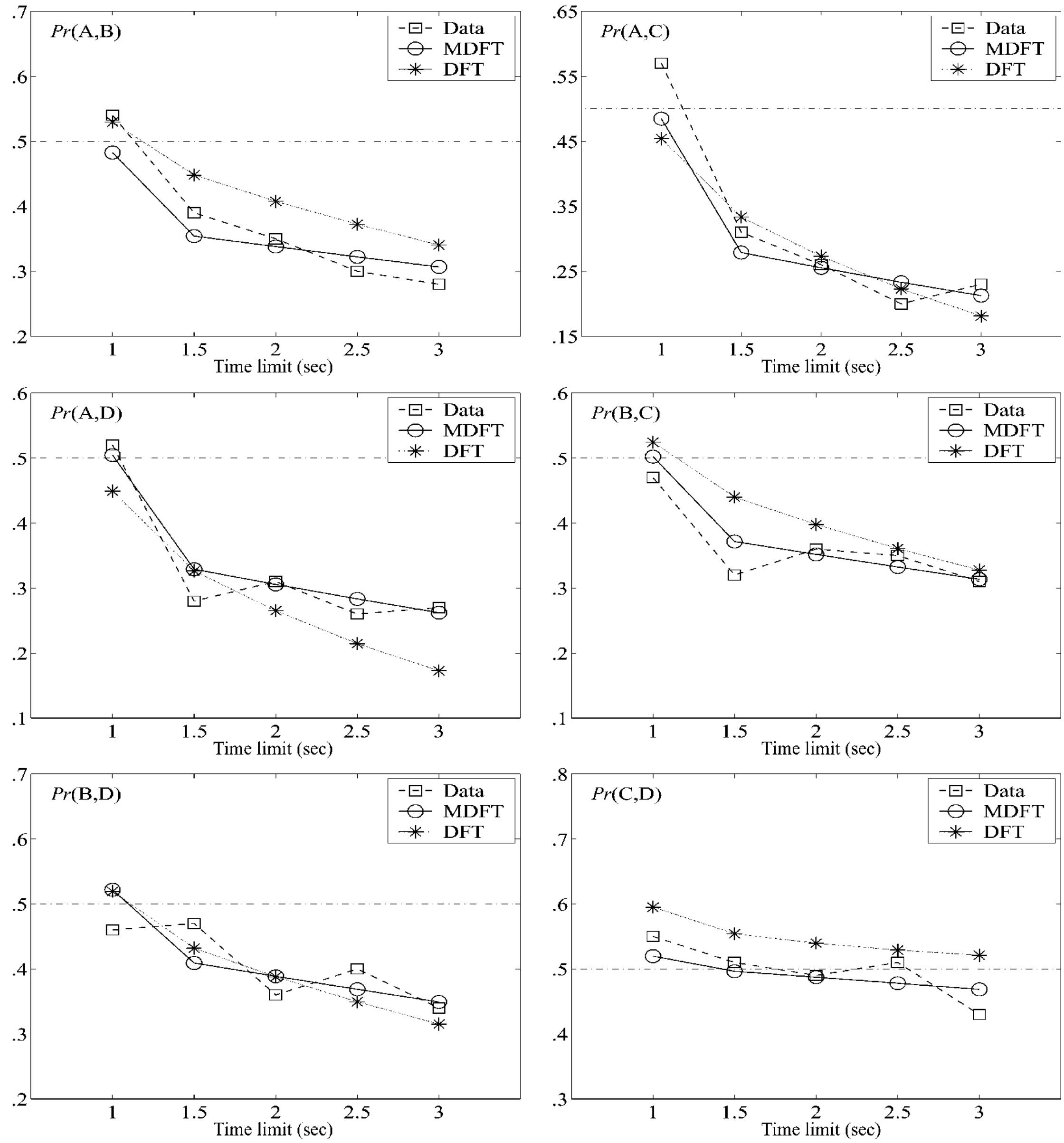

Figure 3. Observed (squares) and predicted (circles for multiattribute decision field theory [MDFT], stars for decision field theory [DFT]) choice probabilities for six choice pairs with five time limits for 1 participant. Both models can predict preference reversals as a function of imposed time limits.

most salient attribute determining choice. (2) Decision criteria were symmetric—that is, no a priori bias was assumed for any alternative, and they were an increasing function of the decision time. (3) The rate of switching from one attribute to another was time invariant. (4) To further reduce the number of parameters of the model, the parameters of the distributions for generating the alternatives were used as fixed parameters of the model. That is, when two alternatives were compared with respect to a given attribute, the *difference* of the means of the generating distributions was taken and weighted by the model parameters (mean valence) that had to be estimated from the data. To account for the finding that losses were weighted differently than gains, two different weights were assumed for constituting the mean valence for the money attribute. For example, a comparison of alternatives A and C with respect to money gave the following equation for determining the mean valence: $\delta_1 = a_1 \cdot (-2) - a_2 \cdot (1)$, where a_1 was a weight for losses, (-2) the expected money for choosing A, a_2 a weight for gains, and $(+1)$ the expected money for choosing C. Furthermore, a comparison of these alternatives with respect to duration gave $\delta_2 = a_3 \cdot$

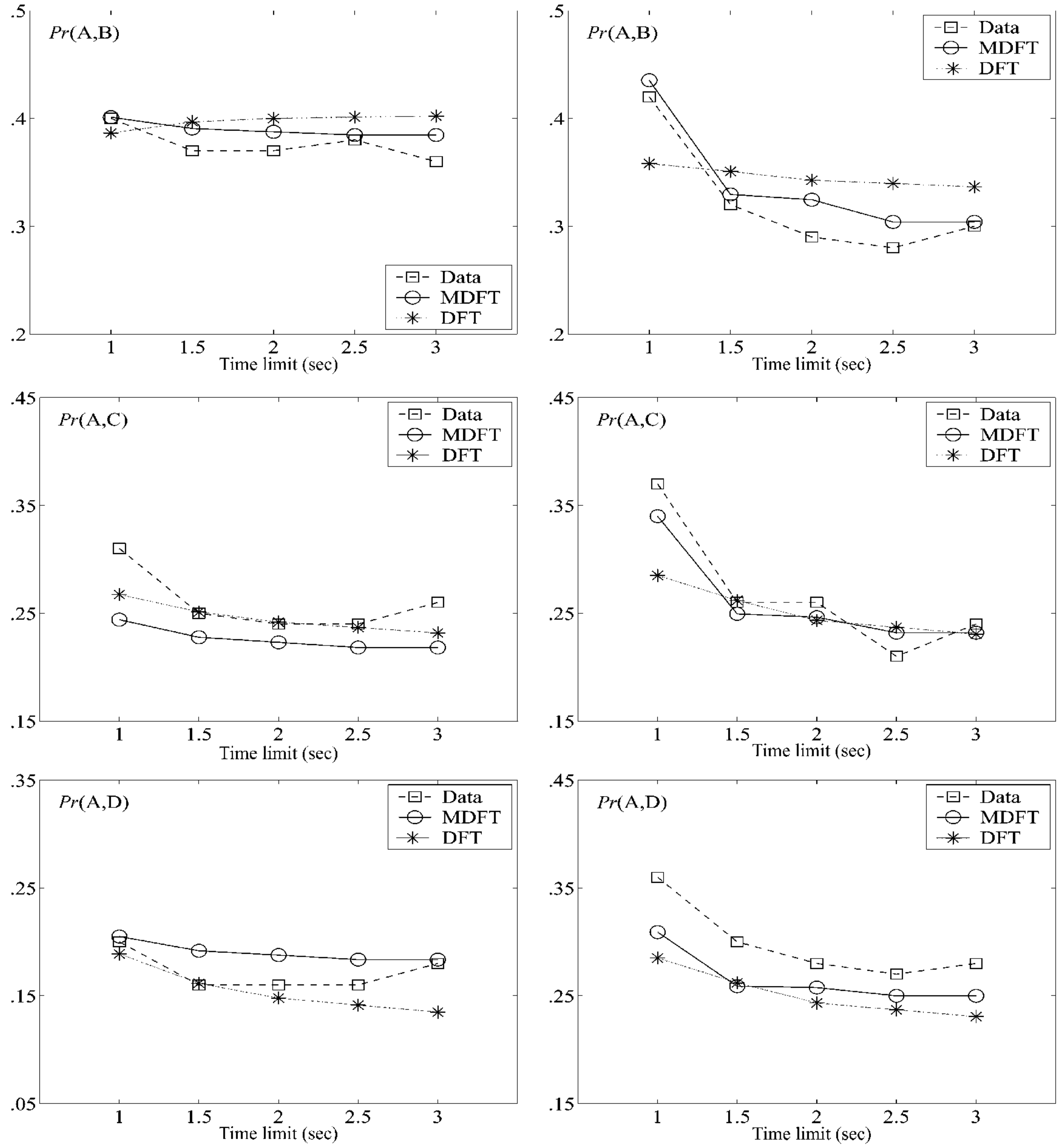

Figure 4. Observed (squares) and predicted (circles for multiattribute decision field theory [MDFT], stars for decision field theory [DFT]) choice probabilities for six choice pairs (AB, AC, AD, BC, BD, CD) with five time limits. The panels on this page refer to the women's group, consisting of 6 participants; those on the next page refer to the men's group, consisting of 5 participants.

(-6), where (-6) was the difference between the expected noise for A and C. a_1, a_2, and a_3 were estimated from the data. Further details are found in Table A1 in the Appendix.

Altogether, there were nine parameters to be estimated from the data (30 conditions, 600 [500] observations for each condition): two parameters, a_1 (loss) and a_2 (gain), to determine the mean valence when two alternatives were compared with respect to the money attribute and one parameter, a_3, to determine the mean valence when two alternatives were compared with respect to the duration attribute; five decision criteria, θ_1, θ_2, θ_3, θ_4, and θ_5, for the five different time limits; and $w_{ij} = w_{ji}$, the attention-switching rate, for switching from the money attributes (i) to the duration attribute (j).

MDFT is the multiattribute extension of DFT. The latter was developed for unidimensional choice alternatives and describes the decision process by a single valence. For applying it to multiattribute alternatives, the mean valence for DFT is assumed to be the sum of the valences for each attribute (cf. Roe et al., 2001)—for example, comparing alternatives A and C gives $\delta = a_1 \cdot (-2) + a_2 \cdot (-1) + a_3 \cdot$

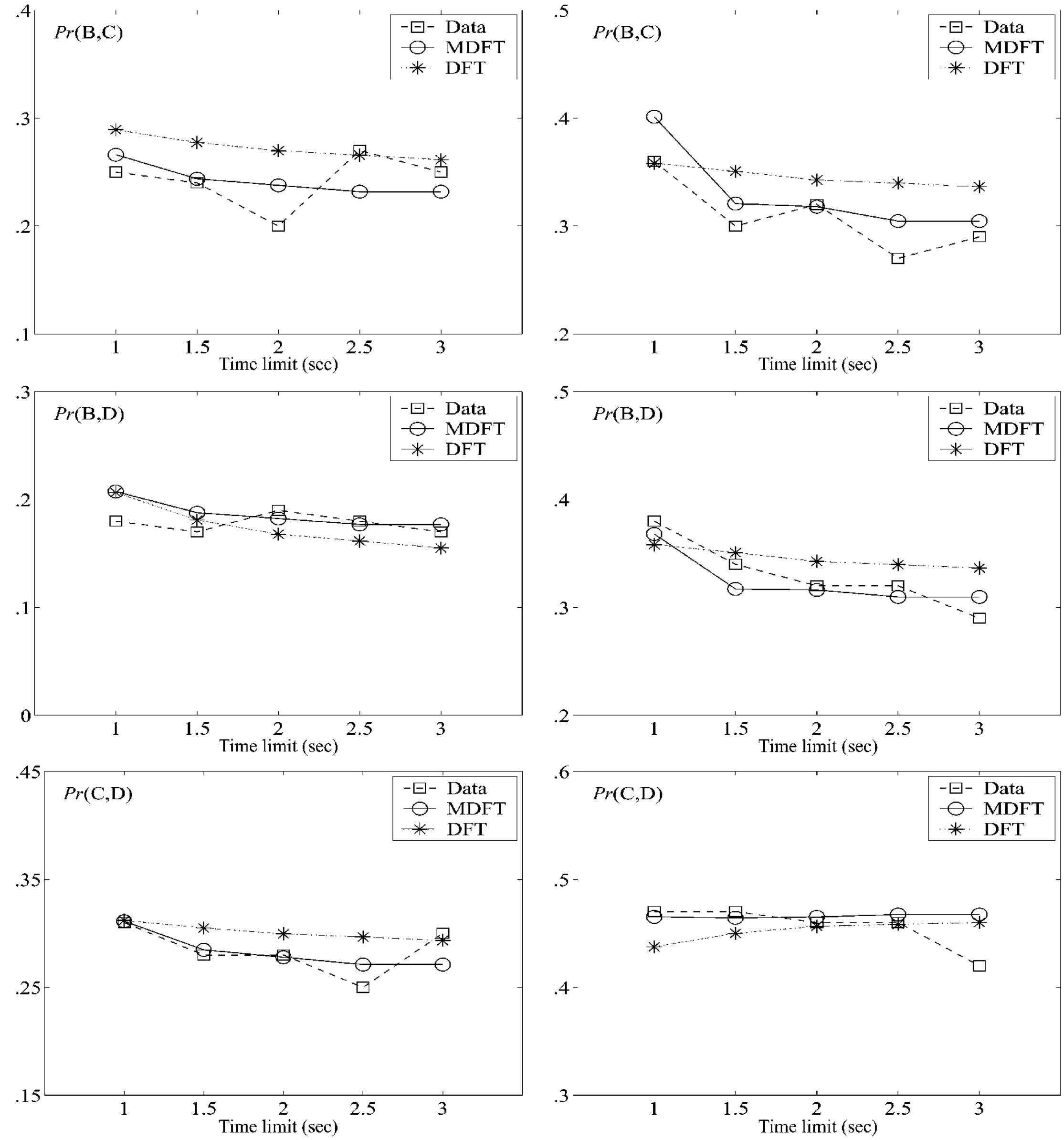

Figure 4 (Continued).

(−6) (for further details, see the Appendix, Table A1). Note that DFT can predict preference reversals (or substantial changes in preference) under time pressure only if a bias toward the less preferred alternative is assumed. Therefore, this bias is included as a model parameter. Thus, the number of parameters to be estimated from the data is the same for both models. The parameters were estimated by minimizing the sum of squared deviations of the observed and predicted choice probabilities, using the FMINSEARCH routine of MATLAB (MDFT (DFT): .0172 [.0224] for the women group; .171 [.0495] for the men group). The estimated parameter values are found in Table A3 in the Appendix.

The data and the fits of MDFT and DFT are presented in Figure 3. The left panels refer to the women, the right to the men. All the figures have the same probability range; the actual scale values differ according to the observed choice probabilities. As was mentioned, the female group did not change preferences as much as the male group as a function of time limits. MDFT and DFT captured the qualitative *pattern* about equally well, but obviously, the fits were not identical. (The sum of squared errors are given in the Appendix.) For the male group, the fits were quite different. In particular, consider the choice pairs AB, AC, AD, BC, and BD. MDFT nicely accounted for the sharp decrease of the choice probabilities from a time limit of 1 sec to a time limit of 1.5 sec. DFT always predicted a monotonic (decreasing or increasing) relationship between choice probabilities and time limits, whereas MDFT could also account for the nonmonotonicity observed in some of the data.

Note that strong restrictions were imposed on the models and, therefore, on the data in fitting the models. Nevertheless, the models were quite successful in accounting for the observed choice pattern. Allowing more freedom across the choice pairs would, of course, improve the fit.

SUMMARY AND DISCUSSION

Decision making under time pressure is a familiar experience. Research on judgment and decision making under time pressure indicates that the probability for choosing an alternative changes as a function of time constraints. Basically, two theoretical approaches have been proposed to account for this observation. One approach assumes that the decision maker is provided with a collection of decision strategies (decision rules, decision heuristics). Depending on the decision situation, the decision maker decides which decision strategy is appropriate. Theories differ with respect to situation/person determinants for selecting a strategy (e.g., Beach & Mitchell, 1978; Ben Zur & Breznitz, 1981; Janis & Mann, 1977; Payne, Bettman, & Johnson, 1988; see Edland & Svenson, 1993, for a review). The second approach assumes that the decision maker considers and compares attributes of choice alternatives sequentially over time. The process of comparison (preference process) stops as soon as enough evidence has been accumulated for either of the alternatives, and then a decision is made. MDFT, as presented in this paper, belongs to the latter class of models. Rather than changing a decision strategy, the decision maker changes a decision criterion, depending on time constraints. In particular, the decision criterion is assumed to be an increasing function of the time limit. Thus, these models provide a parsimonious way to account for preference changes depending on context.

MDFT accounts for decision making under uncertainty for multiattribute binary choice alternatives. Although the deliberation process is neither directly observable nor open to introspection, some important properties of the model can be deduced: The preference for alternatives A and B may change during the deliberation process; the preference strength may depend on the particular attribute; the decision may depend on the order in which the attributes are processed; the decision may depend on how long each attribute is processed and, most important for this study, on how much time is available to make a decision. A numerical example illustrated these predictions. MDFT is the only model that can predict monotonic and nonmonotonic relationships between choice and imposed time limits, qualitatively and quantitatively, as well as preference reversal as a function of the available decision time, simply by invoking the concept of attention switching from one attribute to another during deliberation time. Without time pressure, the decision maker switches back and forth between attributes until enough preference for either of the alternatives has been accumulated. Under time pressure, however, the decision maker has to stop earlier and cannot consider all attributes or reconsider attributes.

For testing the model, choice alternatives with two attributes, money and duration of an annoying sound, were designed. The values of each attribute were drawn from normal distributions and were learned prior to the decision experiment. The decision maker made a decision on the basis of the remembered and anticipated values of the attributes. Note that these experiments differ from experiments in sequential decision making, where the decision maker decides to take another observation or to stop (e.g., Diederich, 2001; Dror et al., 1999). In the former, the single steps are not observable, whereas in the latter they are. In the former, emphasis is put on the cognitive deliberation process; in the latter, on how many observations are taken.

Five men and 6 women participated in the experiment. Most participants changed their preferences for at least some choice alternatives, depending on the imposed time limit. For all the participants, the amount of money to be gained seemed to be the most important attribute for making a choice. For all the participants, nonmonotonic relationships between the imposed time limits and the choice probabilities could be observed. MDFT can account for different individual choice patterns depending on the person's attitude. Here, the data were grouped by gender. The women changed their preferences as a function of time limit to a less extent than did the men.

Although strong restrictions were imposed on the model and, thereby, on the data, MDFT could account for the complex choice patterns observed in the data. MDFT was compared with the predictions of DFT, which considers unidimensional choice alternatives. DFT (Busemeyer & Townsend, 1993) requires an a priori bias for the less preferred alternative in order to predict preference reversals as a function of time limits. Including the bias parameter in the estimation procedure leads to the same number of parameters being estimated for both MDFT and DFT. The predictions of both models, however, are qualitatively different. DFT always predicts monotonic choice probabilities as a function of time limits, whereas MDFT can also predict a nonmonotonic relation depending on attention switching between attributes that are opposite with respect to favoring the alternatives. This feature seems to be unique to MDFT.

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NOTES

1. Multiattribute decision field theory was first introduced under the label MDFT in Diederich (1995; see also Diederich & Busemeyer, 1999). However, there should be no confusion when considered in the respective context.

2. The full version of the model is $P(t + h) = (1 - h \cdot \gamma_i)\, P(t) + V_i(t + h)$, where γ determines the growth or decay of the preference process and is related to the distinction between approach ($\gamma < 0$) and avoidance ($\gamma > 0$) conflicts (see, e.g., Busemeyer & Townsend, 1993; Diederich, 2003; Diederich & Busemeyer, 1999). For small time unit h, the process will approximate a continuous time process. For simplicity's sake, γ is set to zero—that is, no conflict, and h is set to 1.

3. Note that preference reversals, as in this example, are not necessarily predicted by MDFT. Depending on which attribute the decision maker attends first, the length of time for which he or she considers it influences the change in preference as a function of decision time (see below and Diederich, 1997, for details).

4. The switching rates, in this example, are chosen so as to demonstrate clearly the changes of probabilities as a function of time limits for the given δs and for a relatively small range of time limits. Note that the attention-switching rate may be larger for larger δs in order to predict changes of choice probabilities as a function of time.

5. This method, commonly used in psychophysics, has recently been proposed to be applied in decision-making studies (e.g., Payne, Bettman, & Schkade, 1999).

(*Continued on next page*)

APPENDIX

A comparison of two alternatives with respect to attribute parameters of Table 1 results in difference values as shown in Table A1. For example, comparing A and B with respect to money results in $(-2) - (-1) = -1$ for losses and no gains; comparing A and C with respect to money results in (-2) for losses and (1) for gains. For the estimation procedure, y_{i3} was divided by two to be in the same range as y_{i1} and y_{i2}.

For MDFT, the mean valence for the money attribute for choice pair i, $i = 1, \ldots, 6$ is determined as $\delta_{i1} = a_1 \cdot y_{i1} - a_2 \cdot y_{i2}$. The respective mean valence for duration attribute is $\delta_{i2} = a_3 \cdot y_{i3}$, where the ys refer to the numbers in Table A1. Note that only a_1, a_2, and a_3 in these equations are estimated.

For DFT, the mean valence for choice pair i, $i = 1, \ldots, 6$ is determined as $\delta_i = a_1 \cdot y_{i1} - a_2 \cdot y_{i2} + a_3 \cdot y_{i3}$.

Testing for equality of several probabilities (see Table A2), the statistic

$$v = n \cdot \left(\sum_{j=1}^{m} \sum_{k=1}^{r} \frac{h_{jk}^2}{h_j h_k} - 1 \right)$$

is χ^2 distributed with $(m-1)(r-1)$ df, where n is the number of observations, m the number of different categories, r the number of classes within each category, and h denotes the respective frequencies within the classes and categories. Here, $m = 5$ (time limits) and $r = 2$ (binary choice), $\chi^2(4) = 9.488$, $p = .95$.

Table A1
Comparison (Difference) Values for All Choice Pairs, i = 1,2,3,4,5,6 With Respect to (Aspects of) Attributes j = 1,2,3

Choice pairs and Index (i)	Comparison Values (y_{ij}) for Money		Duration
	y_{i1}	y_{i2}	y_{i3}
AB, 1	−1	0	−2
AC, 2	−2	1	−6
AD, 3	−2	2	−8
BC, 4	−2	1	−4
BD, 5	−2	2	−6
CD, 6	0	1	−2

Table A2
v Values of Testing for Equality of Several Probabilities

Gender	Choice Pair: AB	AC	AD	BC	BD	CD
Women	1.7	10.5	6.0	7.8	1.0	6.6
Men	27.9	38.5	12.8	10.6	9.8	1.6

Table A3
Estimated Parameter Values

Model	Gender	a_1	a_2	a_3	w_{ij}	Bias	θ_1	θ_2	θ_3	θ_4	θ_5
MDFT	women	0.0608	0.0880	−0.0415	0.0247	–	15.49	18.00	19.17	20.26	20.29
	men	0.0716	0.0394	−0.0373	0.0191	–	2.28	17.75	18.81	22.89	22.90
DFT	women	0.0534	0.0629	−0.0503	–	−2.56	17.26	20.60	23.28	24.11	25.48
	men	0.0522	0.0419	−0.0419	–	−2.37	16.40	19.89	22.75	23.58	24.95

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